Senary summary so far

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Abstract

This paper attempts to summarise the discussion of senary, or base-6, numeral systems in this and the preceding issue of LT *and to place the results for New Guinea in a wider typological perspective. In light of current crosslinguistic evidence about numeral bases, to be distinguished into construction-bases and cycle-bases, some universals on record need to be adjusted, while a couple of new universals can be suggested specifically for base-6 systems.*

Keywords: *number systems, numeral base, numerals*

It used to be believed that senary (or base-6) numeral systems do not exist: "there is no historical record of a base 6 numbering system anywhere in the world" (Ifrah 1998: 91). The chapter on numeral bases in *The world atlas of language structures* (Comrie 2005) or the survey of numeral rara by Hammarström (2007) indeed do not specifically mention base-6 systems. The most extensive linguistically-informed documentation of numerals to date, Chan 2009, does not specifically acknowledge their existence, either. However, the supposed universal that numeral systems can only be binary (base-2), quaternary (base-4), quinary (base-5), octonary (or octal, base-8, see Avelino 2006), decimal (base-10), duodecimal (base-12), vigesimal (base-20), sexagesimal (base-60), mixed (especially vigesimal-decimal), or restricted (limited to something between 3 and around 20, and with no cyclically recurring base) has now been faulted by Donohue (2008), Hammarström (2009), and Evans (2009). Documenting the same or similar languages of New Guinea, similar counterevidence had been adduced by Lean (1992: Chapter 5) and on the website of GLEC 2004. Thus, in this respect the range of typological variation is wider than had been believed: there are unlikely to be, or also to have been, a

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great many speech communities which structure their numeral systems around base-6; but some clearly do.

What remains is the explanatory task of accounting for their comparative rarity.

The values designated as bases in numeral systems could be arbitrary or they could be motivated by the cognitive and cultural purposes which numbers serve in speech communities – counting objects, measuring quantities, and performing calculations. Abstract mathematics does not uniquely favour or rule out particular choices of bases, and evolutionary simulations like Hurford's (1999) do produce a wide range of choices. However, strong favourites are generally assumed to result from the association of numbers (and the linguistic expressions for them) with the human body and in particular the five fingers of each hand. This association, whether it has the nature of a visual aid or a conceptual model, is widely assumed to account for the crosslinguistic prevalence of base-10, followed by base-20 and base-5. (See Heine 1997: Chapter 2, for a recent summary.)

Now, as Donohue (2008) and Evans (2009) show, base-6 can equally be motivated through body-part tallying. They can point to real finger-counting practices in the South New Guinean speech communities concerned; earlier, Kewitsch (1904) had to resort to conjecture when he sought a motive for the salience of 6 and 60 in Sumerian (or indeed pre-Sumerian) mathematics, previously explained in terms of astronomy:[1]

> Die linke Hand erhebt je einen Finger, man erhält die Zahlen 1 bis 5. Darauf mit dem Rufe "Sechs" erhebt man den ersten Finger der rechten Hand, zugleich schliesst sich die linke Hand. Während der erste Finger der rechten Hand aufrecht bleibt, zählt man weiter von neuem die Finger der linken Hand und erhält so $6+1$ (7), $6+2(8)\ldots6+5(11)$. Darauf hebt man den zweiten Finger der rechten Hand mit dem Ruf 2 Sechs (12). Nach 5 Sechs $+5(35)$ käme 6 Sechs $= 6^2 = 36$, wofür ein besondrer Name steht; diese Zal wird durch den Kleinzeh des linken Fusses versinnlicht. Die 5 Zehe des rechten Fuses liefern die Zahlstufe $6^3 = 216$. Andere Gliedmassen des Körpers könnten die weiteren Potenzen von 6 andeuten, wenn man die Hände andrer Personen nicht zur Verfügung hat.
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> So die natürliche Entstehung des 6-Systems. Leider kann ich ein lebendes Volk nicht nennen, das so zählt, wie ich es eben dargelegt habe. (Kewitsch 1904: 87–88, "Reihenzählung")
>
> Während die einzelnen Finger der linken Hand wie beim ersten Verfahren die 5 Einer bedeuten, kann die ganze ausgebreitete Hand das Sinnbild für 6 sein. Die

1. Kewitsch suggested that the historical Sumerian system was the result of a mixture of prehistoric base-10 and base-6 systems; but this prehistory was to remain in the dark, despite continuing speculations about substrates, superstrates, and adstrates in this and neighbouring areas.

> Finger der rechten Hand geben dann die Haufen $6 \cdot 1$, $6 \cdot 2$, $6 \cdot 3$, $6 \cdot 4$, $6 \cdot 5$; Handhand ist $6 \cdot 6 = 6^2 = 36$ wäre dann ebenfalls die höhere Stufe. (Kewitsch 1904: 91, "Haufenzählung")

Base-8 in Pamean has also been motivated somatically, with the knuckles of both hands (thumbs excluded) rather than fingers as counters (Avelino 2006: 51).[2] Thus, in light of what humans can do with their bodies when they count and perform arithmetic, the extreme rarity of certain numeral bases, including 6, remains unaccounted for.

In addition to the ubiquitous and perennial availability of the body-model, which is more versatile than sometimes believed, it presumably needs some specially strong cultural motivation for 6 to gain prominence as a base in numeral systems, to the exclusion of or also in addition to other body-based bases. As Hammarström (2009) and Evans (2009) suggest, in line with earlier ethnological observations, this motivation in Southern New Guinea is to do with the tuber-growing economy, and in particular with the ritual counting of yams (two-times-three by two-times-three), and their six-by-six arrangement for storage. Points of debate are whether this kind of motive has diffused areally or actual numeral systems have been borrowed, or whether speech communities can get something as out-of-the-ordinary as a base-6 system only from their own ancestors, with the sharing of this trait thus indicative of genealogical relatednesss of the languages concerned.

In order to determine the role of stimulus diffusion, form borrowing, or inheritance, we should have some idea of what the incidence of base-6 numeral systems really is. There are in fact areas or families (or reconstructed proto-languages spawning families) other than Southern New Guinea for which such systems have been assumed or hypothesised:

(i) Sumerian, an extinct isolate of Mesopotamia, famously had base-60, with 10 and 6 sometimes claimed as auxiliary bases (e.g., by Ifrah 1998: Chapter 8);

(ii) some Niger-Congo languages have occasionally been reported as having 6 as one of their bases, usually in addition to 10 or 20 (Zaslavsky 1999: Section 2, Chapters 3 and 4, inspired by Schmidl 1915 and reproducing Schmidl's map with nine triangles for supposed base-6 systems scattered over West and Central Africa);

(iii) reconstructions of Proto-Finno-Ugric or Proto-Uralic have sometimes been interpreted as suggestive of a base-6 system, on the assumption that

2. Elsewhere, base-8 has been attributed to a counting of the spaces between the fingers. Less plausibly, real or claimed polydactyly among shamans has been suspected as a motive for supposed base-6 systems in proto-Finno-Ugric or proto-Uralic. Supernumerary fingers or toes are a rare congenital physical anomaly, and its incidence would not seem to show any correlation with demonstrated base-6 numeral systems.

numerals up to 6 can be reconstructed for the proto-language (with this hypothesis reviewed and rejected in Honti 1993, 1999, on the grounds of other numerals such as 10 and 20 being reconstructible too, not indicative of base-6, and of 9 and 8 being formed subtractively from base-10, with only 7 remaining as a borrowing);

(iv) likewise on the strength of possible reconstructions of proto-forms, ending at 6, Costanoan and Miwok, two distantly related families which together make up the Utian family of Central California, are assumed to have had base-6, and base-6 is assumed to have been an areal trait with traces also in Yokutsan (Beeler 1961; Gamble 1980; Callaghan 1990, 1994; Blevins 2005).

In fact, when surveying numeral systems, from a comprehensive and reliable compilation such as Chan 2009, one not infrequently encounters some sort of a break after 6, with 6 a basic form and one or two subsequent numerals constructed ($7 = 6+1$, $8 = 6+2$, $9 = 6+3$, which is the most widespread relevant pattern; or $6 = 5+1$, $7 = [5+1]+1$, $8 = [5+1]+2$, $9 = [5+1]+3$ as in Miskito, Misumalpan family, or in Onjob, Trans-New Guinea; or $7 = 10-3$, $8 = 10-2$, $9 = 10-1$ as in Yapese, an Oceanic language within Austronesian), or with numerals 1 through 6 native and (some) subsequent numerals borrowed (as in Indo-Aryan Romani all over Europe; dialects of Mundari in South India; non-Austronesian Bunak of East Timor); or one finds 6 being utilised in the construction of one or another non-consecutive higher numeral (as in Breton, of the Celtic family within Indo-European, where $18 = 3 \times 6$).

The question about a base universal and its validity, then, turns into one of how to define "bases" in numeral systems.

First, it should be clear that linguistic universals are about linguistic forms and constructions, about lexicons and grammars, and not about mathematics. And this is not perforce the same thing in the present domain. Thus, while a good case can presumably be made for Sumerian and Babylonian mathematics being based on 60, 10, and 6, the Sumerian numerals were as follows (Thomsen 1987: 82):

1	*diš*, *dili*, *aš*		
2	*min*		
3	*eš*$_5$		
4	*limmu*		
5	*iá*		
6	*àš*	= *iá* + *aš*	$5+1$
7	*imin*	= *iá-min*	$5+2$
8	*ussu*	= *iá-eš*$_5$	$5+3$
9	*ilimmu*	= *iá-limmu*	$5+4$

10	*u*
20	*niš*
30	*ušu*$_2$
40	*nimin*, *nin*$_5$
50	*ninnu*
60	*g̃íš*, *g̃éš*
3600	*šár*

Evidently, the numerals for 6 through 9 were built on that for 5 (a quinary system in this sense) and there is no evidence for the numeral for 6 cyclically recurring in numerals for multiples of 6 or exponentiation with base 6.

Further, in discussions of numeral systems which are intended as dealing with linguistic expressions, two basic concepts of "base" can be distinguished: a CONSTRUCTION-base and a CYCLE-base. Numerals are frequently referred to as "bases" when they are an atomic expression (or at any rate not transparently compositional, synchronically speaking) and when expressions for other numerals are formally based on them, with higher or lower numerals constructed by arithmetic operations (addition, subtraction, multiplication) with their help. The requirement of such a construction-base being itself atomic is sometimes waived – in which case a numeral for 6, analysable into $5+1$, can be called a base (as it is in Hurford 1999) when subsequent numerals include it, with $7=[5+1]+1$ etc. as in Miskito. If only a single other numeral is constructed from another numeral through a given arithmetic operation, like $18=3\times 6$ in Breton or $12=2\times 6$ in Mankanya (Atlantic, Niger-Congo; Zaslavsky 1999), this would almost seem too little to qualify as a construction-base. A cycle-base is the narrower concept: a numeral is a cycle-base if it is a construction-base and cyclically recurs in linguistic designations of multiples of the respective number (base-6: 6, 12, 18, 24, ...) and/or in exponentiation with that base (base-6: 6, 36, 216, ...). Comrie (2005: 530) is among those subscribing to a narrow definition of this kind: "By the 'base' of a numeral system we mean the value *n* such that numeral expressions are constructed according to the pattern ... $xn + y$, i.e. some numeral *x* multiplied by the base plus some other numeral" – which explains why the *WALS* sample lacks relevant languages.[3]

On current evidence, construction-base-6 would not seem uncommon, with its incidence widespread and not circumscribed in terms of families or areas. What Pott (1868: 3) and many others after him observed about the diachronic instability of decimal, vigesimal, and quinary systems would seem to be even more valid for construction-base-6:

3. Hammarström 2007 too looked for cycle-bases 6, but only Hammarström 2009 found some.

> Welche der [...] drei Zählmethoden übrigens gewisse Völker befolgen mögen [...]: das entscheidet über das STAMMVERHÄLTNISS der gerade in Frage stehenden Völker zu einander, als davon unabhängige und zuweilen sogar sich damit entzweiende Erscheinung, wenig oder – nichts.

By contrast, cycle-base-6 indeed appears to be rare, and so far only the Southern New Guinea languages figuring in the present discussion in *LT* are reliably described convincing cases. A possible further base-6 system where the base is at least rudimentarily cyclic could be that of "Bolan", a language of Guinea sketched by Zaslavsky (1999), possibly the Atlantic, Niger-Congo language with Northern Bullom or Bullom So as alternate names: according to Zaslavsky, the system is essentially quinary, except that $12 = 6 \times 2$ (as in Mankanya) and (going beyond Mankanya) $24 = 6 \times 4$; no information is provided about 18. But otherwise the African languages which have here and there been mentioned as showing 6 as one of their bases, Niger-Congo and other, are not cycling on 6 at all and merely have special additive constructions for 7, 8, and possibly 9 – or indeed up to 12 in Balanta-Kentohe (Atlantic, Niger-Congo; Zaslavsky 1999[4]) – based on 6 or a special multiplicative construction for 12 (2×6).

With the rarum status of cycle-base-6 confirmed by present evidence, explanations in terms of prerequisites such as stimulus diffusion, form borrowing, or inheritance, or in terms of whatever else could conceivably account for being rare (that is, for not having been innovated often, and/or for having frequently and rapidly been abandoned when once innovated) remain high on the typological agenda. I will conclude this summary by offering a diachronic speculation that would account for cycle-base-6 in essentially the same terms as for construction-base-6, seeing both as elaborations of restricted numeral systems.

Restricted numeral systems are not uncommon, and, although seriously endangered, continue to be found far apart in Africa, Australia and Oceania, South and Meso-America, and perhaps beyond (Comrie 2005). When limited to around 5, they usually involve finger-counting, and the meanings or at any rate etymologies of numerals are to do with the hand and its parts and with actions of the hands accompanying counting. The highest numeral in such systems is likely to be, or to be derived from, an approximate quantifier (paucal 'a few' or multal 'many') or the universal quantifier ('all') – in the spirit of a universal suggested by Stampe (1976: 559, No. 1511 in the Konstanz UNIVERSALS ARCHIVE (Plank et al. 2009)). Now, when the highest precise numeral is 5, as counted on all fingers of one hand and thereby exhausting one entire hand, the subsequent numeral would be one comprehending anything beyond 5 – that is, as long as the system remains restricted: once it gets extended, '6 plus' (or 'a

4. Chan 2009, however, has contemporary Balanta-Kentohe as purely decimal.

few, many, all') would be a natural designation of 6 and a natural starting point for continuing with the numeral system. As summarised above, a construction-base-6, with numerals after 6 formed compositionally with constituents 6 and other available numerals, indeed is not uncommon, and is something that does not seem to need special cultural encouragement or the direct model of one's own elders or of another speech community one is in contact with. Also, etymologies of numerals for 6 would often seem to fit in with such a pivotal position at the threshold from a restricted to an extended system: 'all', 'whole', 'beyond five fingers', 'fist' are etyma that have been plausibly suggested (e.g., Christian 1957, Blevins 2005).

The recognition of 6 as a threshold implies a corresponding modification of some universals on record which set the limit at 5. Thus, Stampe's (1976: 597) universal about the provenance of a highest numeral needs to make allowances for numeral systems consisting of less than six numerals (rather than five). Also, Greenberg's (1978: 256) universal about "simple lexical systems" of numerals, which are ones not involving arithmetic operations, should be corrected from 5 to 6 as the largest value (No. 530 in THE UNIVERSALS ARCHIVE).

With construction-base-6 and a moderately extended numeral system, there is nothing that would prevent the recycling of base-6 for the expression of values that are multiples or power sets of 6 – given that precise numerals are required for such large quantities. On present evidence, it seems that few cultures and societies have taken this momentous step, entirely left to their own devices and with no inspiration from unrestricted systems that would have served as a model or material. But arguably this is the only way to get to cycle-base-6, suitably encouraged. "No numeral systems with bases $6 \times 1, 6 \times 2, 6 \times 3, \ldots$ or $6^1, 6^2, 6^3, \ldots$ without yams" stands as an achronic universal, to complement this diachronic law.

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