A superlative analysis of superlative scalar modifiers

1. Background and Overview

- Recent analyses of comparative and superlative quantifiers as alternatives to Generalized Quantifier Theory (Hackl 2001, 2009; Geurts & Nouwen 2007; Nouwen 2008, 2010)
- Modal effects arising with the superlative scalar modifiers *at least/ at most*
- In this talk:
 - empirical observations about at least/ at most carry over to at the earliest/ at the latest
 - analysis of at the earliest/ at the latest most in terms of superlative semantics
 - need for modal interpretation follows from presuppositions of the superlative
 - extension of the analysis to at least/ at most

2. Data and observations

- In most contexts, *at least/ at most* imply speaker uncertainty, i.e. they imply that the speaker isn't sure about the precise value (see Geurts & Nouwen 2007):
- a. John had at least five beers last night.b. I have at least/ at most three children.
- This implication of speaker uncertainty vanishes if *at least/at most* combine with certain modals (see Geurts & Nouwen 2007):
 - *at least* with necessity modals
 - *at most* with possibility modals
- (2) a. Your cabin luggage can/may weigh at most 10 kg.b. You must be at least 2m tall to become a basketball player.
- The implication of speaker uncertainty also vanishes in certain generic contexts (cf. Nouwen 2010):
- (3) Computers of this kind have at least/at most 2GB of memory.
- Geurts & Nouwen (2007) take the modal component to be part of the lexical meaning of *at least/at most;* rule of modal concord to make it disappear when embedded under the appropriate modal.
- Nouwen (2010) argues that an epistemic possibility modal is inserted to rescue utterances involving *at least/at most* that would otherwise be ruled out due to there being a simpler expression with the same meaning available.
- The observations for *at least/at most* carry over to the temporal adverbials *at the earliest/ at the latest*:

(4)	a. John left at midnight at the latest/at the earliest.	speaker uncertainty
	b. You may leave at 6 pm at the earliest.	no speaker uncertainty
	c. You must leave at 6 pm at the latest.	no speaker uncertainty
	d. On weekends, John gets up at 8 am at the latest/ at the earliest	no speaker uncertainty

• The parallel between *at least /at most* and *at the earliest /at the latest* suggests that the observed pattern is due to superlative morphology and semantics.

3. Superlatives: Truth Conditions and Presuppositions (Heim 1995/99)

- Subject (external argument) of superlative morpheme is an individual (type e)
- (5) $[[-est]](C)(R_{\langle d, \langle e, t \rangle \rangle})(x_e) \text{ is defined only if } (i) x \in C$ $(ii) \forall y [y \in C \rightarrow \exists d R(d)(y)]$ $(iii) \exists y [y \in C \& y \neq x]$ If defined,

 $[[-est]](C)(R_{<d,<e,t>>})(x_e) = 1 \text{ iff } \exists d [R(d)(x) \& \forall y [y \in C \& y \neq x \to \neg R(d)(y)]]$

Ad presupposition (i):

(6) All of these candidates are acceptable. But John is most impressive. (Heim 1995/99)
 ⇒ John is one of these candidates.

Ad presupposition (iii):

(7) #You are the best mother I have. (Hackl 2009)
Example derivation
(8) Peter kam am spätesten. (German) Peter arrived at-the latest 'Peter was the last to arrive.'

The LF is derived by QR of the external argument followed by parasitic QR of *-est_C* in between:

(9) Peter -est_C $\lambda d \lambda x$ [PAST s* [$\langle i,t \rangle$ d-late] [$\langle i,t \rangle \lambda t_1 x$ arrive t_1]]]

The adverbial *d-late* combines with the temporal abstract of the VP via Predicate Modification.

(10) a. $[[PAST]] = \lambda t'_i \cdot \lambda P_{it} \cdot \exists t [t < t' \& P(t)]$ b. $[[late]] = \lambda d_i \cdot \lambda t_i \cdot t \ge d$ c. $[[arrive]] = \lambda t_i \cdot \lambda x_e \cdot x \text{ arrives at } t$

(11) a. truth conditions: ∃d [∃t [t < s* & t ≥ d & Peter arrives at t] & ∀y [y ∈ C & y ≠ Peter → ¬∃t' [t' < s* & t' ≥ d & y arrives at t']]] b. presuppositions:

- (i) Peter $\in C$
- (ii) $\forall y [y \in C \rightarrow \exists d \exists t [t < s^* \& t \ge d \& y \text{ arrives at } t]$ (i.e. all persons in C arrived at some time)

(iii) $\exists y [y \in C \& y \neq Peter]$

C is made up of persons who arrived at some time, e.g. $C = \{Ann, Bill, Peter\}$

(12) $t_a = t_b = t_p = s^*$

4. Temporal superlative modifiers: spätestens 'at the latest' / frühestens 'at the earliest'

- Semantics of *spät* 'late' and *früh* 'early' (cf. von Stechow 2009):
- (13) a. [[late]] = λd_i . λt_i . $t \ge d$ b. [[early]] = λd_i . λt_i . t < d
- External argument of superlative morpheme is a time (type i)
- $[-est](C)(R_{\langle d, \langle i, t \rangle \rangle})(t_i)$ is defined only if (i) $t \in C$ (14)(ii) $\forall t' [t' \in C \rightarrow \exists d R(d)(t')]$ (iii) $\exists t' [t' \in C \& t' \neq t]$ If defined, $[[-est]](C)(R_{<d,<i,t>>})(t_i) = 1 \text{ iff } \exists d [R(d)(t) \& \forall t' [t' \in C \& t' \neq t \to \neg R(d)(t')]]$ (15) Peter kam frühestens um 6 Uhr. (German) Peter arrived at-the-earliest at 6 o'clock $[[at]] = \lambda t_i \cdot \lambda t'_i \cdot \lambda P_{\langle i,t \rangle} \cdot t' = t \& P(t')$ (16)(17)LF: 6 o'clock < d,< i,t >> -est_C λd λt_2 PAST < i,t > < i,t > < i,t > d early λt_3

<< i,t > ,t >

ti

< i,t >

Peter arrived t₁

 λt_1

at t_2

(18) a. truth conditions:

 $\exists d [\exists t [t < s^* \& t < d \& t = 6 o'clock \& Peter arrives at t] \&$

 $\forall t' [t' \in C \& t' \neq 6 \text{ o'clock} \rightarrow \neg \exists t''[t'' \leq s^* \& t'' \leq d \& t''=t' \& \text{Peter arrives at } t'']]$ b. presuppositions:

- (i) 6 o'clock \in C
- (ii) $\forall t [t \in C \rightarrow \exists d \exists t' [t' < s^* \& t' < d \& t' = t \& Peter arrives at t']$
- (i.e. all times in C are times before the speech time at which Peter arrived) (iii) $\exists t [t \in C \& t \neq 6 \text{ o'clock }]$
- C is made up of times at which Peter arrived (in the contextually relevant interval)
- The presuppositions (18b) (ii) and (iii) together entail that Peter arrived at more than one time (in the contextually relevant interval). Thus LF (14) leads to a presupposition failure.
- Presupposition failure can be avoided if a covert epistemic modality operator is inserted at LF (see Nouwen 2010). Sentence (15) is then equivalent to (19).
- (19) Peter kann frühestens um 6 Uhr gekommen sein. (German) Peter can at-the-earliest at 6 o'clock arrived have
- (20) a. [[have]] = $\lambda t'_{i.}\lambda P_{it.}\exists t [t < t' \& P(t)]$ b. [[can]] = $\lambda t_{i.}\lambda P_{s(it)}$. $\exists w' \in Acc(w,t)$: P(w')(t)
- (21) a. truth conditions: ∃d [∃w' ∈ Acc(w,s*): ∃t [t < s* & t < d & t = 6 o'clock & Peter arrives in w' at t] & ∀t' [t' ∈ C & t' ≠ 6 o'clock → ¬∃w'' ∈ Acc(w,s*): ∃t'' [t'' < s* & t'' < d & t'' = t' & Peter arrives in w' at t'']]]
 b. presuppositions: (i) 6 o'clock ∈ C (ii) ∀t [t ∈ C → ∃d ∃w' ∈ Acc(w,s*): ∃t' [t < s* & t' < d & t' = t & Peter arrives in

w' at t']

(i.e. all times in C are times at which Peter arrived in some epistemically accessible world)

- (iii) $\exists t [t \in C \& t \neq 6 \text{ o'clock }]$
- C is made up of times at which Peter arrives in some epistemically accessible world.
- The presuppositions in (ii) and (iii) can be fulfilled, as Peter's arrival time might be different in different epistemically accessible worlds.



5. At least / at most as superlatives

- at most = $much_A + -est$
- at least = $little_A + -est$
- (23) adverbial *much* / *little*: a. [[much_A]] = λd_d . $\lambda d'_d$. $d' \ge d$ b. [[little_A]] = λd_d . $\lambda d'_d$. d' < d
- External argument of superlative morpheme is a degree (type d)
- (24) $[[-est]](C)(R_{\langle d, \langle d, t \rangle \rangle})(d_d) \text{ is defined only if } (i) \quad d \in C$ $(ii) \quad \forall d' [d' \in C \rightarrow \exists d'' R(d'')(d')]$ $(iii) \quad \exists d' [d' \in C \& d' \neq d]$

If defined, [[-est]] (C)($R_{<d,<d,t>>}$)(d_d) = 1 iff $\exists d$ ''[$R(d'')(d) \& \forall d'$ [$d' \in C \& d' \neq d \rightarrow \neg R(d'')(d')$]]

- (25) John is at most 2m tall.
- (26) Derivation of LF:
 - a. The DegP 2m is QRed out of the AP: 2m $\lambda d'$ [John d'-tall]
 - b. -est-much is merged countercyclically in between $2m [-est_c much_A] \lambda d' [$ John d'-tall]
 - c. -est is QRed to a position below DegP:
- (27)



- (28) $[[tall]] = \lambda d_d$. λx_e . Height(x) $\ge d$
- (29) a. truth conditions: ∃d [2m ≥ d & Height(j) ≥ 2m & ∀d' [d' ∈ C & d' ≠ 2m → ¬ [d'≥ d & Height(j) ≥ d']]] b. presuppositions: (i) 2m ∈ C (ii) ∀d' [d' ∈ C → ∃d [d'≥ d & Height(j) ≥ d'] (iii) ∃d [d ∈ C & d ≠2m]

- The truth conditions (29a) say that John is exactly 2m tall. The presuppositions (29b) can be fulfilled if C consists of degrees below John's height.
- Assuming a functional meaning of adjectives as in (30), rather than a relational one as in (28), a presupposition failure results. The presuppositions (31b) (ii) and (iii) together entail that John has more than one height.
- (30) $[[tall]] = \lambda d_d$. λx_e . Height(x) = d
- (31) a. truth conditions:
 ∃d [2m ≥ d & Height(j) = 2m &
 ∀d' [d' ∈ C & d' ≠ 2m → ¬ [d'≥ d & Height(j) = d']]]
 b. presuppositions:
 - (i) $2m \in C$
 - (ii) $\forall d' [d' \in C \rightarrow \exists d [d' \ge d \& \text{Height}(j) = d']$
 - (iii) $\exists d [d \in C \& d \neq 2m]$
- Sensible presuppositions and correct truth conditions result when combined with a possibility modal.
- (32) John can be at most 2m tall.

(33)



(34) a. truth conditions:

 $\exists d [2m \ge d \& \exists w' \in Acc(w,s^*): Height_{w'}(j) = 2m \&$

 $\forall d' [d' \in C \& d' \neq 2m \rightarrow \neg [d' \geq d \& \exists w'' \in Acc(w,s^*): \text{Height}_{w''}(j) = d']]]$ b. presuppositions:

- (i) $2m \in C$
- (ii) $\forall d' [d' \in C \rightarrow \exists d [d' \geq d \& \exists w' \in Acc(w,s^*): Height_{w'}(j) = d']$
- (iii) $\exists d [d \in C \& d \neq 2m]$

6. Generalizing the semantics of *at least / at most*

- Generalize the meaning ascribed to *at least / at most* to cover also cases not involving gradable adjectives:
- (35) a. Bill is at least an associate professor.
 - b. Mary climbed at least K2.
 - c. John got as least as far as Rome.
- generalized adverbial *much* / *little* applying to a property and involving a contextually supplied ranking function μ :
- (36) a. $[[much_A]] = \lambda d_d$. $\lambda P_{\langle e,t \rangle}$. $\mu(P) \ge d$ b. $[[little_A]] = \lambda d_d$. $\lambda P_{\langle e,t \rangle}$. $\mu(P) < d$
- There must be at least one property in the alternatives that is not implied (see also literature on *only*).

Consider a scenario where John is on a car trip through Italy with destination Naples. He follows a designated route going through Milano, Florence and Rome.

(37) ------ Milano Florence Rome Naples

- (38) a. John got as least as far as Rome.b. #John got as least as far as Naples.
- (39) $[[-est]](C)(R_{\langle d, \langle \langle e, t \rangle \rangle})(P_{\langle e, t \rangle}) \text{ is defined only if (i)} P \in C$ (ii) $\forall Q [Q \in C \rightarrow \exists d R(d)(Q)]$ (iii) $\exists Q [Q \in C \& P \not\subset Q]$ If defined,
 - $\llbracket-\text{est}\rrbracket(C)(R_{<d,<<e,t>})(P_{<e,t>}) = 1 \text{ iff } \exists d[R(d)(P) \& \forall Q [Q \in C \& P \not\subset Q \rightarrow \neg R(Q)(d)]]$
- The condition that there be alternative properties not implied also allows using a relational meaning of gradable adjectives and derive the infelicity of LF (27) as presupposition failure.

7. Summary

- The observation that *at least / at most* share certain behaviour with *at the earliest / at the latest* suggests that it is due to superlative semantics.
- It is possible to give a fully compositional analysis of *at least / at most* in terms of superlative semantics.
- The fact that *at least / at most* are illicit without an epistemic possibility modal follows from the presuppositions of the superlative.

8. Directions for Further Work

- The superlative analysis replicates the findings of Nouwen (2010) in a fully-compositional way based on superlative semantics. The same problem with necessity modals arises.
- For combinations with necessity modals correct truth conditions can't be derived (see Nouwen 2010).

- (40) Peter muss spätestens um 6 Uhr gekommen sein. (German) Peter must at-the-latest at 6 o'clock arrived have
- (41) a. truth conditions:
 - $\exists d [\forall w' \in Acc(w,s^*): \exists t [t < s^* \& t \ge d \& t = 6 \text{ o'clock } \& \text{ Peter arrives in } w' at t] \& \\ \forall t' [t' \in C \& t' \ne 6 \text{ o'clock} \rightarrow \neg \forall w'' \in Acc(w,s^*): \exists t'' [t'' < s^* \& t'' \ge d \& \\ t'' = t' \& \text{ Peter arrives at } t'']]]$

b. presuppositions:

- (i) 6 o'clock \in C
- (ii) ∀t[t∈C→∃d∀w'∈Acc(w,s*): ∃t' t< s* & t' ≥ d & t'=t & Peter arrives in w' at t']
 (i.e. all times in C are times at which Peter arrived in all epistemically accessible world)

(iii) $\exists t [t \in C \& t \neq 6 \text{ o'clock }]$

- The truth conditions (41a) aren't correct for two reasons. They say that:
- Peter arrived at 6 o'clock in <u>all</u> epistemically accessible worlds.

- Peter didn't arrive later in <u>all</u> worlds, i.e. there might be some worlds in which he arrived later. Moreover, the presuppositions (ii) and (iii) are contradictory.

- The same problem arises for *at least* combined with necessity:
- (42) John must be at least 2m tall.
- (43) a. truth conditions: ∃d [2m ≥ d & ∀w' ∈ Acc(w,s*): Height_{w'}(j) = 2m & ∀d' [d' ∈ C & d' ≠ 2m → ¬ [d'≥ d & ∀w'' ∈ Acc(w,s*): Height_{w''}(j) = d']]]
 b. presuppositions: (i) 2m ∈ C (ii) ∀d' [d' ∈ C → ∃d [d'≥ d & ∀w' ∈ Acc(w,s*): Height_{w'} (j) = d'] (iii) ∃d [d ∈ C & d ≠2m]
- We would get correct truth conditions and presuppositions, if *must* were analyzed as a possibility modal, i.e. ∃ instead of ∀ (Nouwen's solution).
- But this would not explain that *at least / at the latest* combine happily with necessity modals, but not with possibility modals.

(44)	a. John can/may be at most 2m tall.b. John can/may be at least 2m tall.	no speaker uncertainty speaker uncertainty
(45)	a. John must be at least 2m tall.b. John must be at most 2m tall.	no speaker uncertainty speaker uncertainty
(46)	a. You may leave at 6 pm at the earliest.b. You may leave at 6 pm at the latest.	no speaker uncertainty speaker uncertainty
(47)	a. You must leave at 6 pm at the latest.b. You must leave at 6 pm at the earliest.	no speaker uncertainty speaker uncertainty

• In the present analysis, *at most* and *at the latest* are based on the positive polar expression (*much* and *late*). But *at most* patterns with *at the earliest* in combining happily with possibility modals, while *at the latest* patterns with *at least*.

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