A superlative analysis of superlative scalar modifiers

1. Background and Overview

- Modal effects arising with the superlative scalar modifiers at least/ at most
- In this talk:
  - empirical observations about at least/ at most carry over to at the earliest/ at the latest
  - analysis of at the earliest/ at the latest most in terms of superlative semantics
  - need for modal interpretation follows from presuppositions of the superlative
  - extension of the analysis to at least/ at most

2. Data and observations

- In most contexts, at least/ at most imply speaker uncertainty, i.e. they imply that the speaker isn’t sure about the precise value (see Geurts & Nouwen 2007):
  (1)  a. John had at least five beers last night.
       b. I have at least/ at most three children.
  - This implication of speaker uncertainty vanishes if at least/at most combine with certain modals (see Geurts & Nouwen 2007):
    • at least with necessity modals
    • at most with possibility modals
  (2)  a. Your cabin luggage can/may weigh at most 10 kg.
       b. You must be at least 2m tall to become a basketball player.
  - The implication of speaker uncertainty also vanishes in certain generic contexts (cf. Nouwen 2010):
  (3)  Computers of this kind have at least/at most 2GB of memory.
  - Geurts & Nouwen (2007) take the modal component to be part of the lexical meaning of at least/at most; rule of modal concord to make it disappear when embedded under the appropriate modal.
  - Nouwen (2010) argues that an epistemic possibility modal is inserted to rescue utterances involving at least/at most that would otherwise be ruled out due to there being a simpler expression with the same meaning available.
  - The observations for at least/at most carry over to the temporal adverbials at the earliest/ at the latest:
a. John left at midnight at the latest/at the earliest.

b. You may leave at 6 pm at the earliest.

c. You must leave at 6 pm at the latest.

d. On weekends, John gets up at 8 am at the latest/at the earliest

- The parallel between at least/at most and at the earliest/at the latest suggests that the observed pattern is due to superlative morphology and semantics.


- Subject (external argument) of superlative morpheme is an individual (type e)

\[
[[\text{-est}]](C)(R_{d,l,e,p}\tau)(x_e) \text{ is defined only if}
\]

(i) \( x \in C \)

(ii) \( \forall y [y \in C \rightarrow \exists d R(d)(y)] \)

(iii) \( \exists y [y \in C \& y \neq x] \)

- If defined,

\[
[[\text{-est}]](C)(R_{d,l,e,p}\tau)(x_e) = 1 \text{ iff } \exists d [R(d)(x) \& \forall y [y \in C \& y \neq x \rightarrow \neg R(d)(y)]]
\]

Ad presupposition (i):

(6) All of these candidates are acceptable. But John is most impressive. (Heim 1995/99) 

\( \Rightarrow \) John is one of these candidates.

Ad presupposition (iii):

(7) You are the best mother I have. (Hackl 2009)

- Example derivation

(8) Peter kam am spätesten. (German)

Peter arrived at the latest

‘Peter was the last to arrive.’

The LF is derived by QR of the external argument followed by parasitic QR of -estC in between:

(9) Peter -estC \( \lambda d \lambda x [PAST s^* [<_t, p \rightarrow d-late] [<_t, p \rightleftharpoons \lambda t \_x \_arrive t]] \)

The adverbial d-late combines with the temporal abstract of the VP via Predicate Modification.

(10) a. \( [[PAST]] = \lambda t_1 \lambda P_1 \exists t [t < s^* \& P(t)] \)

b. \( [[\text{late}]] = \lambda d_1 \lambda t_1, t \geq d \)

c. \( [[\text{arrive}]] = \lambda t_1 \lambda x_e \_x \_arrive t \)

(11) a. truth conditions:

\[ \exists d [\exists t [t < s^* \& t \geq d \& \_x \_arrive t] \& \forall y [y \in C \& y \neq \_x \_Peter \rightarrow \neg \exists t' [t' < s^* \& t' \geq d \& y \_arrive t']] ] \]

b. presuppositions:

(i) \( \_x \_Peter \in C \)

(ii) \( \forall y [y \in C \rightarrow \exists d \exists t [t < s^* \& t \geq d \& y \_arrive t] \)

(i.e. all persons in C arrived at some time)

(iii) \( \exists y [y \in C \& y \neq \_x \_Peter] \)

C is made up of persons who arrived at some time, e.g. C = {Ann, Bill, Peter}

(12) \[ \_t_a \_t_b \_t_p \_s^* \]
4. Temporal superlative modifiers: spätestens ‘at the latest’ / frühestens ‘at the earliest’

- Semantics of spät ‘late’ and früh ‘early’ (cf. von Stechow 2009):
  (13)  
  a. $[[\text{late}]] = \lambda d. \lambda t_i. t \geq d$
  b. $[[\text{early}]] = \lambda d. \lambda t_i. t < d$

- External argument of superlative morpheme is a time (type i)
  (14)  
  $[[\text{-est}}](C)(R_{d,<i,t>})(t_i)$ is defined only if 
  (i) $t \in C$
  (ii) $\forall t' [ t' \in C \rightarrow \exists d R(d)(t')]$
  (iii) $\exists t' [ t' \in C \land t' \neq t ]$

  If defined, 
  $[[\text{-est}}](C)(R_{d,<i,t>})(t_i) = 1 \iff$ 
  $\exists d [ R(d)(t) \land \forall t' [ t' \in C \land t' \neq t \rightarrow \neg R(d)(t') ] ]$

(15)  
Peter kam frühestens um 6 Uhr. (German)
Peter arrived at-the-earliest at 6 o’clock

(16)  
$[[\text{at}]] = \lambda t_i. \lambda t'_i. \lambda P_{<i,t>}. t' = t \land P(t')$

(17)  
LF:

6 o’clock

-est C

$\lambda d$

$\lambda t_2$

PAST

$< i, t >$

$< i, t >$

d = early

$\lambda t_3$

$t_3$

at $t_2$

$\lambda t_1$

Peter arrived $t_1$
(18)  a. truth conditions:
    \[ \exists d \left[ \exists t \left[ t < s^* \& t < d \& t = 6 \text{ o'clock} \& \text{Peter arrives at } t \right] \& \right. \\
    \left. \forall t' \left[ t' \in C \& t' \neq 6 \text{ o'clock} \rightarrow \neg \exists t'' \left[ t'' < s^* \& t'' < d \& t'' = t' \& \text{Peter arrives at } t'' \right] \right] \]

    b. presuppositions:
    (i) 6 o’clock \ \in \ C
    (ii) \ \forall t \left[ t \in C \rightarrow \exists d \exists t' \left[ t' < s^* \& t' < d \& t' = t \& \text{Peter arrives at } t' \right] \right]
        (i.e. all times in C are times before the speech time at which Peter arrived)
    (iii) \ \exists t \left[ t \in C \& t \neq 6 \text{ o'clock} \right]

    • C is made up of times at which Peter arrived (in the contextually relevant interval)
    • The presuppositions (18b) (ii) and (iii) together entail that Peter arrived at more than one
      time (in the contextually relevant interval). Thus LF (14) leads to a presupposition failure.
    • Presupposition failure can be avoided if a covert epistemic modality operator is inserted at
      LF (see Nouwen 2010). Sentence (15) is then equivalent to (19).

(19)  Peter kann frühestens um 6 Uhr gekommen sein.  (German)
      Peter can at-the-earliest at 6 o’clock arrived have

(20)  a. [have] = \lambda t_i. \lambda P_i. \exists t \left[ t < t' \& P(t) \right] 
    b. [can] = \lambda t_i. \lambda P_{it(i)}. \exists w' \in \text{Acc}(w,t): P(w')(t)

(21)  a. truth conditions:
    \[ \exists d \left[ \exists w' \in \text{Acc}(w,s^*): \exists t \left[ t < s^* \& t < d \& t = 6 \text{ o’clock} \& \text{Peter arrives in } w' \text{ at } t \right] \& \right. \\
    \left. \forall t' \left[ t' \in C \& t' \neq 6 \text{ o’clock} \rightarrow \neg \exists w'' \in \text{Acc}(w,s^*): \exists t'' \left[ t'' < s^* \& t'' < d \& t'' = t' \& \text{Peter arrives in } w' \text{ at } t'' \right] \right] \]

    b. presuppositions:
    (i) 6 o’clock \ \in \ C
    (ii) \ \forall t \left[ t \in C \rightarrow \exists d \exists w' \in \text{Acc}(w,s^*): \exists t' \left[ t < s^* \& t' < d \& t' = t \& \text{Peter arrives in } w' \text{ at } t' \right] \right]
        (i.e. all times in C are times at which Peter arrived in some epistemically accessible
         world)
    (iii) \ \exists t \left[ t \in C \& t \neq 6 \text{ o’clock} \right]

    • C is made up of times at which Peter arrives in some epistemically accessible world.
    • The presuppositions in (ii) and (iii) can be fulfilled, as Peter’s arrival time might be different
      in different epistemically accessible worlds.
5. At least / at most as superlatives

- at most = much_A + -est
- at least = little_A + -est

(23) adverbial much / little:
   a. \([\text{much}_A] = \lambda d. \lambda d' \cdot d' \geq d\)
   b. \([\text{little}_A] = \lambda d. \lambda d' \cdot d' < d\)

- External argument of superlative morpheme is a degree (type d)

(24) \([-\text{est}]_C (R_{<d,<d,t>})(d_d)\) is defined only if
   (i) \(d \in C\)
   (ii) \(\forall d' \cdot [d' \in C \rightarrow \exists d'' R(d'')(d')]\)
   (iii) \(\exists d' \cdot [d' \in C \& d' \neq d]\)

If defined,
\([-\text{est}]_C (R_{<d,<d,t>})(d_d) = 1 \text{ iff } \exists d'' \cdot [R(d'')(d) \& \forall d' \cdot [d' \in C \& d' \neq d \rightarrow \neg R(d'')(d')]\]

(25) John is at most 2m tall.

(26) Derivation of LF:
   a. The DegP 2m is QRed out of the AP:
      \(2m \lambda d' \cdot \text{John } d'\text{-tall}\)
   b. \(-\text{est}-\text{much}\) is merged countercyclically in between
      \(2m \cdot [-\text{est}_C \text{ much}_A] \lambda d' \cdot \text{John } d'\text{-tall}\)
   c. \(-\text{est}\) is QRed to a position below DegP:

(27)

(28) \([\text{tall}] = \lambda d, \lambda x. \text{Height}(x) \geq d\)

(29) a. truth conditions:
   \(\exists d \cdot [2m \geq d \& \text{Height}(j) \geq 2m \& \forall d' \cdot [d' \in C \& d' \neq 2m \rightarrow \neg [d' \geq d \& \text{Height}(j) \geq d']]\]
   b. presuppositions:
      (i) \(2m \in C\)
      (ii) \(\forall d' \cdot [d' \in C \rightarrow \exists d \cdot [d' \geq d \& \text{Height}(j) \geq d']\)
      (iii) \(\exists d \cdot [d \in C \& d \neq 2m]\)
• The truth conditions (29a) say that John is exactly 2m tall. The presuppositions (29b) can be fulfilled if C consists of degrees below John’s height.

• Assuming a functional meaning of adjectives as in (30), rather than a relational one as in (28), a presupposition failure results. The presuppositions (31b) (ii) and (iii) together entail that John has more than one height.

(30) \[ [\text{tall}] = \lambda d. \lambda x. \text{Height}(x) = d \]

(31) a. truth conditions:
   \[ \exists d \ [ 2m \geq d & \text{Height}(j) = 2m & \forall d' [ d' \in C & d' \neq 2m \rightarrow \neg [ d' \geq d & \text{Height}(j) = d' ]] ] \]

b. presuppositions:
   (i) \[ 2m \in C \]
   (ii) \[ \forall d' [ d' \in C \rightarrow \exists d [ d' \geq d & \text{Height}(j) = d'] ] \]
   (iii) \[ \exists d [ d \in C & d \neq 2m ] \]

• Sensible presuppositions and correct truth conditions result when combined with a possibility modal.

(32) John can be at most 2m tall.

(33)

(34) a. truth conditions:
   \[ \exists d \ [ 2m \geq d & \exists w' \in \text{Acc}(w,s*): \text{Height}_{w'}(j) = 2m & \forall d' [ d' \in C & d' \neq 2m \rightarrow \neg [ d' \geq d & \exists w'' \in \text{Acc}(w,s*): \text{Height}_{w''}(j) = d' ] ] ] \]

b. presuppositions:
   (i) \[ 2m \in C \]
   (ii) \[ \forall d' [ d' \in C \rightarrow \exists d [ d' \geq d & \exists w' \in \text{Acc}(w,s*): \text{Height}_{w'}(j) = d' ] ] \]
   (iii) \[ \exists d [ d \in C & d \neq 2m ] \]
6. Generalizing the semantics of *at least / at most*

- Generalize the meaning ascribed to *at least / at most* to cover also cases not involving gradable adjectives:
  
  (35) a. Bill is at least an associate professor.
  
  b. Mary climbed at least K2.
  
  c. John got as least as far as Rome.

- Generalized adverbial *much / little* applying to a property and involving a contextually supplied ranking function $\mu$:
  
  (36) a. $[[\text{much}_A]] = \lambda d. \lambda P_{<e,t}>. \mu(P) \geq d$
  
  b. $[[\text{little}_A]] = \lambda d. \lambda P_{<e,t}>. \mu(P) < d$

- There must be at least one property in the alternatives that is not implied (see also literature on *only*).

  Consider a scenario where John is on a car trip through Italy with destination Naples. He follows a designated route going through Milano, Florence and Rome.

  (37) ---|---|---|---|---|---|---
  
  Milano Florence Rome Naples

(38) a. John got as least as far as Rome.
  
  b. #John got as least as far as Naples.

(39) $[[\text{-est}}](C)(R_{<d,<e,t,>})(P_{<e,t,>})$ is defined only if

(i) $P \in C$

(ii) $\forall Q [Q \in C \rightarrow \exists d R(d)(Q)]$

(iii) $\exists Q [Q \in C & P \notin Q]$

If defined,

$[[\text{-est}}](C)(R_{<d,<e,t,>})(P_{<e,t,>}) = 1$ iff $\exists d [R(d)(P) & \forall Q [Q \in C & P \notin Q \rightarrow \neg R(Q)(d)]]$

- The condition that there be alternative properties not implied also allows using a relational meaning of gradable adjectives and derive the infelicity of LF (27) as presupposition failure.

7. Summary

- The observation that *at least / at most* share certain behaviour with *at the earliest / at the latest* suggests that it is due to superlative semantics.

- It is possible to give a fully compositional analysis of *at least / at most* in terms of superlative semantics.

- The fact that *at least / at most* are illicit without an epistemic possibility modal follows from the presuppositions of the superlative.

8. Directions for Further Work

- The superlative analysis replicates the findings of Nouwen (2010) in a fully-compositional way based on superlative semantics. The same problem with necessity modals arises.

- For combinations with necessity modals correct truth conditions can’t be derived (see Nouwen 2010).
Peter muss spätestens um 6 Uhr gekommen sein. (German)
Peter must at-the-latest at 6 o’clock arrived have

(41) a. truth conditions:
   \[ \exists d \ [ \forall w' \in \text{Acc}(w, s^*): \exists t \ [ t < s^* \land t \geq d \land t = 6 \text{ o’clock} \land \text{Peter arrives in } w' \text{ at } t ] \land \forall t' \ [ t' \in C \land t' \neq 6 \text{ o’clock} \implies \neg \exists t'' \in \text{Acc}(w', s^*): \exists t''' \ [ t''' < s^* \land t''' \geq d \land t''' = t' \land \text{Peter arrives at } t'' ] \] \]

b. presuppositions:
   (i) 6 o’clock \in C
   (ii) \forall t \ [ t \in C \implies \exists d \ [ d \in C \land d = 6 \text{ o’clock} ] \]
   (i.e. all times in C are times at which Peter arrived in all epistemically accessible world)
   (iii) \exists t \ [ t \in C \land t \neq 6 \text{ o’clock } ]

- The truth conditions (41a) aren’t correct for two reasons. They say that:
  - Peter arrived at 6 o’clock in all epistemically accessible worlds.
  - Peter didn’t arrive later in all worlds, i.e. there might be some worlds in which he arrived later. Moreover, the presuppositions (ii) and (iii) are contradictory.

- The same problem arises for at least combined with necessity:

(42) John must be at least 2m tall.

(43) a. truth conditions:
   \[ \exists d \ [ 2m \geq d \land \forall w' \in \text{Acc}(w, s^*): \text{Height}_{w'}(j) = 2m \land \forall d' \ [ d' \in C \land d' \neq 2m \implies \neg \exists d \ [ d \geq d' \land \forall w'' \in \text{Acc}(w, s^*): \text{Height}_{w''}(j) = d' ] \] \]

b. presuppositions:
   (i) 2m \in C
   (ii) \forall d' \ [ d' \in C \implies \exists d \ [ d \geq 2m \land \forall w' \in \text{Acc}(w, s^*): \text{Height}_{w'}(j) = d' ] \]
   (iii) \exists d \ [ d \in C \land d \neq 2m ]

- We would get correct truth conditions and presuppositions, if must were analyzed as a possibility modal, i.e. \( \exists \) instead of \( \forall \) (Nouwen’s solution).
- But this would not explain that at least / at the latest combine happily with necessity modals, but not with possibility modals.

(44) a. John can/may be at most 2m tall. no speaker uncertainty
    b. John can/may be at least 2m tall. speaker uncertainty

(45) a. John must be at least 2m tall. no speaker uncertainty
    b. John must be at most 2m tall. speaker uncertainty

(46) a. You may leave at 6 pm at the earliest. no speaker uncertainty
    b. You may leave at 6 pm at the latest. speaker uncertainty

(47) a. You must leave at 6 pm at the latest. no speaker uncertainty
    b. You must leave at 6 pm at the earliest. speaker uncertainty

- In the present analysis, at most and at the latest are based on the positive polar expression (much and late). But at most patterns with at the earliest in combining happily with possibility modals, while at the latest patterns with at least.
References


