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Some Logical Equivalences & Entailments

(Partee et al. 1990)

1. Laws of Propositional Logic

For any formulas ϕ, ψ, π:

1. Idempotent Laws:	$(\phi \vee \phi)$	$\Leftrightarrow$	ϕ	
	$(\phi \wedge \phi)$	$\Leftrightarrow$	ϕ	
2. Commutative Laws:	$(\phi \vee \psi)$	$\Leftrightarrow$	$(\psi \vee \phi)$	
	$(\phi \wedge \psi)$	$\Leftrightarrow$	$(\psi \wedge \phi)$	
3. Associate Laws:	$((\phi \vee \psi) \vee \pi)$	$\Leftrightarrow$	$(\phi \vee (\psi \vee \pi))$	
	$((\phi \wedge \psi) \wedge \pi)$	$\Leftrightarrow$	$(\phi \wedge (\psi \wedge \pi))$	
4. Distributive Laws:	$(\phi \vee (\psi \wedge \pi))$	$\Leftrightarrow$	$((\phi \vee \psi) \wedge (\phi \vee \pi))$	
	$(\phi \wedge (\psi \vee \pi))$	$\Leftrightarrow$	$((\phi \wedge \psi) \vee (\phi \wedge \pi))$	
5. Identity Laws:	$(\phi \vee \bot)$	$\Leftrightarrow$	ϕ	
	$(\phi \vee \top)$	$\Leftrightarrow$	$\top$	
	$(\phi \wedge \bot)$	$\Leftrightarrow$	$\bot$	
	$(\phi \wedge \top)$	$\Leftrightarrow$	ϕ	
6. Complement Laws:	$(\phi \vee \neg\phi)$	$\Leftrightarrow$	$\top$	
	$(\phi \wedge \neg\phi)$	$\Leftrightarrow$	$\bot$	
	$\neg\neg\phi$	$\Leftrightarrow$	ϕ	
7. DeMorgan's Laws:	$\neg(\phi \vee \psi)$	$\Leftrightarrow$	$(\neg\phi \wedge \neg\psi)$	
	$\neg(\phi \wedge \psi)$	$\Leftrightarrow$	$(\neg\phi \vee \neg\psi)$	
8. Conditional Laws:	$(\phi \rightarrow \psi)$	$\Leftrightarrow$	$(\neg\phi \vee \psi)$	
	$(\phi \rightarrow \psi)$	$\Leftrightarrow$	$(\neg\psi \rightarrow \neg\phi)$	(Contraposition)
9. Biconditional Laws:	$(\phi \leftrightarrow \psi)$	$\Leftrightarrow$	$(\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi)$	
	$(\phi \leftrightarrow \psi)$	$\Leftrightarrow$	$(\neg\phi \wedge \neg\psi) \vee (\phi \wedge \psi)$	

2. (Some) Laws in Predicate Logic

For any predicate π and formula ϕ:

1. Law of Quantifier Negation:
   $\neg\forall x\,(\pi(x)) \quad \Leftrightarrow \quad \exists x\,(\neg\pi(x))$
   [And, by $\neg\neg\phi \Leftrightarrow \phi$, also:
   $\forall x\,(\pi(x)) \quad \Leftrightarrow \quad \neg\exists x\,(\neg\pi(x))$
   $\neg\forall x\,(\neg\pi(x)) \quad \Leftrightarrow \quad \exists x\,(\pi(x))$
   $\forall x\,(\neg\pi(x)) \quad \Leftrightarrow \quad \neg\exists x\,(\pi(x))$]

2. Laws of Quantifier (In)Dependence:
   a. $\forall x\forall y\,(\pi(x,y)) \quad \Leftrightarrow \quad \forall y\forall x\,(\pi(x,y))$
   b. $\exists x\exists y\,(\pi(x,y)) \quad \Leftrightarrow \quad \exists y\exists x\,(\pi(x,y))$
   c. $\exists x\forall y\,(\pi(x,y)) \quad \Rightarrow \quad \forall y\exists x\,(\pi(x,y))$

3. Laws of Quantifier Distribution:
   a. $\forall x\,(\pi(x) \wedge \rho(x)) \quad \Leftrightarrow \quad \forall x\,(\pi(x)) \wedge \forall x\,(\rho(x))$
   b. $\exists x\,(\pi(x) \vee \rho(x)) \quad \Leftrightarrow \quad \exists x\,(\pi(x)) \vee \exists x\,(\rho(x))$
   c. $\forall x\,(\pi(x)) \vee \forall x\,(\rho(x)) \quad \Rightarrow \quad \forall x\,(\pi(x) \vee \rho(x))$
   d. $\exists x\,(\pi(x) \wedge \rho(x)) \quad \Rightarrow \quad \exists x\,(\pi(x)) \wedge \exists x\,(\rho(x))$

4. Laws of Quantifier Movement:
   a. $\phi \rightarrow \forall x(\pi(x)) \quad \Leftrightarrow \quad \forall x\,(\phi \rightarrow \pi(x))$
      provided that x is not free in ϕ.
   b. $\phi \rightarrow \exists x(\pi(x)) \quad \Leftrightarrow \quad \exists x\,(\phi \rightarrow \pi(x))$
      provided that x is not free in ϕ and that someone exists.
   c. $\forall x(\pi(x)) \rightarrow \phi \quad \Leftrightarrow \quad \exists x\,(\pi(x) \rightarrow \phi)$
      provided that x is not free in ϕ and that someone exists.
   d. $\exists x(\pi(x)) \rightarrow \phi \quad \Leftrightarrow \quad \forall x\,(\pi(x) \rightarrow \phi)$
      provided that x is not free in ϕ.